effort required for the two methods about equal. In this regard, working with the eigenvalue equations in statically condensed form (e.g., Eqs. (7) of Ref. 6), so that reduced and eliminated degrees of freedom are coupled only dynamically, would reduce the number of matrix computations required in each step of Paz's dynamic condensation algorithm.

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# Reply by Author to A. H. Flax

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THE writer wishes to thank Flax for his interest and comments on the paper.[19] Flax points out that the dynamic condensation method is a definite improvement over the static condensation method of Guyan[1] (Reference numbers refer to original citations in the paper, Ref. 19) and Irons[2] since the proposed method leads to a virtually exact solution of the generalized eigenproblem that arises in structural dynamics. He also recognizes that the dynamic condensation method improves both the eigenvalues and the eigenvectors and not just the eigenvectors, as is the case with the modified condensation method devised by Kidder,[7] Miller,[14,15] and others. Most important, the dynamic condensation method does not require the series expansion of the term $[[K_{ss}]-\omega^2 [M_{ss}]]^{-1}$ and the truncation of higher order terms, which poses the question of convergence of the series as discussed by Flax in Ref. 9.

Having recognized the merits of the proposed method, Flax states: "However, dynamic condensation greatly increases the amount of computation required, since the method involves repeated inversions of matrices of order equal to the number of degrees of freedom eliminated from the reduced problem." This statement constitutes an unfortunate misunderstanding of the paper and of the method. The dynamic condensation method requires no matrix inversion, but rather the application of the process known as the Gauss-Jordan elimination as it is routinely applied to the solution of a system of linear equations. This elementary process is carried out over the secondary coordinates to be eliminated. As indicated by Eq. (2) of the paper, this process immediately results in the transformation matrix $[\bar{T}]$ and the dynamic matrix $[\bar{D}]$ in Eq. (2). The simple multiplication and addition of matrices then gives the reduced stiffness, Eq. (4), and the reduced mass matrices, Eq. (5). Furthermore, there is no need to perform explicitly the product $[\bar{M}]=[T]^T\ [M]\ [T]$ which involves large matrices of dimensions equal to that of the original problem before reduction. By operating on the partition matrices of the matrices $[T]$ and $[M]$ a simple algorithm is obtained for the direct evaluation of the reduced mass matrix $[\bar{M}]$ which has the dimensions of the reduced problem.

In the illustrative example presented in the Note, the calculation of $[\bar{M}]$ requires the algebraic evaluation of the four elements of the symmetric matrix $[\bar{M}]$. Thus, the example of 48 degrees of freedom reduced to 4 degrees of freedom only requires the sequential applications of the Gauss-Jordan method in the linear symmetric system of 44 equations and the calculation of the elements of $[\bar{K}]$ and $[\bar{M}]$ of order 4 in addition to the eigensolution of four eigenproblems of order 4. These simple calculations result in almost exact eigensolutions for all four modes left in the reduced problem. In the illustrative example, the errors in the four lower eigenvalues obtained after reduction are 1.2%, 0.2%, 0.7%, and 3.5%. These values should be contrasted with the results produced by either static or modified condensation with errors respectively equal to 1.2%, 11.3%, 30.4%, and 36.8%. Furthermore, the application of a single iterative cycle of dynamic condensation results in the virtually exact eigensolution as indicated in Table 1 of the paper.[19]

The writer agrees with Flax in the need for critically comparing dynamic condensation with other methods of eigensolution such as inverse iteration or other variations such as the popular subspace iteration method. However, such comparisons can ony be made after an efficient algorithm is developed for the implementation of the dynamic condensation method. An overview of all the three methods of condensation—static, modified, and dynamic—is given in the previous paper.[17] This material is also presented with numerical examples and corresponding computer program in the second edition of the writer's textbook.[18]

As a final point, considering the fundamental importance that the solution of the eigenproblem has a structural dynamics, the writer agrees with other comments of Flax leading to further development and evaluation of the dynamic condensation method.

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